A class of axisymmetric stationary exact solutions of Einstein-Maxwell equations

1979 J. Phys. A: Math. Gen. 12 383

(http://iopscience.iop.org/0305-4470/12/3/014)

J. Phys. A: Math. Gen., Vol. 12, No. 3, 1979. Printed in Great Britain

A class of axisymmetric stationary exact solutions of Einstein–Maxwell equations

M D Patel

Department of Mathematics, Sardar Patel University, Vallabh Vidyanagar 388 120, India

Received 5 April 1978, in final form 11 August 1978

Abstract. Einstein–Maxwell vacuum field equations of an axially symmetric stationary charged rotating source are studied. A class of asymptotically flat solutions representing the exterior field of a stationary rotating charged oblate spheroidal source is obtained. By examining eigenvectors of Einstein tensor $\mathbf{G}_{ij}$, invariants $R_{ij}R^{ij}$ and Petrov classification of Weyl's conformal curvature tensor, it is proved that the metric potentials describe null electromagnetic fields.

1. Introduction

Astronomical observations show that a large number of heavenly bodies are in a state of stationary rotation about their axes. One of the most interesting effects of rotation is that the radial character of the field is destroyed. Blackett (1947) gave a hypothesis that 'a rotating star produces an electromagnetic field'. The equilibrium shape of a rotating star is an oblate spheroid. The only astronomical objects discovered so far, where general relativistic effects are not negligible, are pulsars. They are dense rotating stars with large magnetic fields. Hence axially symmetric stationary exact solutions of the Einstein–Maxwell equations have attracted much attention. Bonnor (1961) gave a method for generating axially symmetric static electrovac solutions from corresponding solutions of the axisymmetric static Einstein vacuum field equations. Som and Raychaudhari (1968) obtained a cylindrically symmetric solution for charged dust with rotation where, however, the Lorentz force vanishes so that equilibrium is due to the balancing of the gravitational effect of matter and the electromagnetic field energy by the centrifugal action of the rotation. By assuming a linear relation between matter density and magnetic energy, Banerji (1968) obtained a class of cylindrically symmetric static solutions of the Einstein–Maxwell equations.

By considering the metric coefficient g_{00} to be a function of the electrostatic potential A, Synge (1960) obtained solutions which represent the static electrovac universe. In the case of the stationary axisymmetric metric one must consider electromagnetic fields instead of electrostatic fields together with those additional functions appearing in the metric due to presence of rotational asymmetry. Different functional relationships may be assumed depending upon the mathematical expediency or physical motivation. In this paper a class of axially symmetric stationary exact solutions of the Einstein–Maxwell vacuum field equations are obtained. The physical significance of the solution has been discussed by examining eigenvalues and eigenvectors of the Einstein tensor. Further it has been found that the metric is appropriate for the

0305-4470/79/030383+09\$01.00

description of the exterior space–time of an oblate rotating charged spheroid or semi-infinite line source.

2. Metric and field equations

Misra (1960) used oblate spheroidal coordinates to obtain static solutions of empty space–time field equations. Patel (1975) used these coordinates in obtaining axially symmetric zero mass meson solutions of Einstein's equations. As discussed by Patel (1978), the general metric of a stationary axially symmetric space–time in oblate spheroidal coordinates is of the form

$$ds^2 = -e^{2\delta}\,dt^2 + e^{2\sigma}(d\phi - w\,dt)^2 + e^{2\beta}a^2(\theta^2+\alpha^2)\left(\frac{d\theta^2}{1+\theta^2}+\frac{d\alpha^2}{1-\alpha^2}\right) \quad (2.1)$$

where δ, σ, w and β are functions of θ and α with $0 \leqslant \theta < \infty$; $-1 \leqslant \alpha \leqslant 1$. Let us number t, ϕ, θ, α as 0, 1, 2 and 3 respectively. The electromagnetic field tensor arises from two scalar functions A and B as follows (Israel *et al* 1972, Patel 1975)

$$F_{0\mu} = A_{,\mu} \quad (2.2)$$

$$F^{\mu\nu} = \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\lambda}B_{,\lambda} \quad (2.3)$$

where $\epsilon^{\mu\nu\lambda}$ is the Levi–Civita permutation tensor density.

The Einstein–Maxwell field equations are

$$G_j^i = -8\pi E_j^i \quad (2.4)$$

$$4\pi E_j^i = \tfrac{1}{4}g_j^i F^{kl}F_{kl} - F^{ik}F_{jk} \quad (2.5)$$

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \quad (2.6)$$

$$F^{ij}_{;j} = 0 \quad (2.7)$$

where E_j^i is the electromagnetic energy tensor. Roman indices range from 0 to 3, and Greek indices from 1 to 3. A semicolon denotes a covariant differentiation with respect to the metric of space–time, and a comma denotes a partial differentiation.

Now it is easy to see from equations (2.2), (2.3) and (2.5) that $E_2^2 + E_3^3 = 0$.

Hence from equation (2.4)

$$G_2^2 + G_3^3 = 0, \quad (2.8)$$

which implies a simple result

$$\delta + \sigma = k = \text{constant}. \quad (2.9)$$

Then the Einstein–Maxwell (EM) equations (2.4), (2.5) and the Maxwell equations (2.6), (2.7) give the following results.

$$\begin{aligned}
&\Delta\delta - \left(1+\frac{\theta^2-\alpha^2}{2}\right)\Delta\beta - (1+\theta^2)^2\delta_2^2 - (1-\alpha^2)^2\delta_3^2 \\
&\qquad - \tfrac{1}{2}[wP + \tfrac{1}{2}(1-\theta^4)w_2^2 + \tfrac{1}{2}(1-\alpha^4)w_3^2]\exp(2k-4\delta) \\
&\quad = -Q[\{(1+\theta^2)(A_2^2+B_2^2) + (1-\alpha^2)(A_3^2+B_3^2)\}\exp(-2\delta) \\
&\qquad + 2w\sqrt{(1+\theta^2)}\sqrt{(1-\alpha^2)}(A_2B_3 - A_3B_2)\exp(k-4\delta)]
\end{aligned} \quad (2.10)$$

$$(2+\theta^2-\alpha^2)\Delta\beta+2(1+\theta^2)^2\delta_2^2+2(1-\alpha^2)^2\delta_3^2$$
$$=\tfrac{1}{2}[(1+\theta^2)^2 w_2^2+(1-\alpha^2)^2 w_3^2]\exp(2k-4\delta) \tag{2.11}$$

$$\tfrac{1}{2}(\theta^2+\alpha^2)\Delta\beta+(1+\theta^2)^2\delta_2^2-(1-\alpha^2)^2\delta_3^2$$
$$+\tfrac{1}{4}[-(1+\theta^2)^2 w_2^2+(1-\alpha^2)^2 w_3^2]\exp(2k-4\delta)$$
$$=-Q[(1+\theta^2)(A_2^2+B_2^2)-(1-\alpha^2)(A_3^2+B_3^2)]\exp(-2\delta). \tag{2.12}$$

$$P=-4Q^2[w\{(1+\theta^2)(A_2^2+B_2^2)+(1-\alpha^2)(A_3^2+B_3^2)\}\exp(-2\delta)$$
$$+(1+w^2\exp(2k-4\delta))\surd(1+\theta^2)\surd(1-\alpha^2)(A_2B_3-A_3B_2)\exp(-k)] \tag{2.13}$$

$$-\tfrac{1}{2}(1+w^2\exp(2k-4\delta))P+2w\Delta\delta-w\{(1+\theta^2)w_2^2+(1-\alpha^2)w_3^2\}\exp(2k-4\delta)$$
$$=-2\surd(1+\theta^2)\surd(1-\alpha^2)(A_3B_2-A_2B_3)\exp(-k) \tag{2.14}$$

$$2\delta_2\delta_3-\tfrac{1}{2}w_2w_3\exp(2k-4\delta)=-2Q(A_2A_3+B_2B_3)\exp(-2\delta) \tag{2.15}$$

$$2wQ^2\left[\frac{\surd(1+\theta^2)}{\surd(1-\alpha^2)}B_2(w_2-2w\delta_2)+\frac{\surd(1-\alpha^2)}{\surd(1+\theta^2)}B_3(w_3-2w\delta_3)\right.$$
$$\left.+w\{A_2(w_3-2w\delta_3)-A_3(w_2-2w\delta_2)\}\exp(k-2\delta)\right]\exp(3k-6\delta)$$
$$+Q[(1+\theta^2)^{-1/2}(1-\alpha^2)^{-1/2}\{\Delta B-2(1+\theta^2)\delta_2B_2-2(1-\alpha^2)\delta_3B_3\}$$
$$+\{w_3A_2-w_2A_3-4w(\delta_3A_2-\delta_2A_3)\}\exp(k-2\delta)]\exp(k-2\delta)=0. \tag{2.16}$$

$$2wQ[w\surd(1+\theta^2)\surd(1-\alpha^2)\{(w_3-2\delta_3w)B_2-(w_2-2w\delta_2)B_3\}\exp(k-2\delta)$$
$$-\{(1+\theta^2)(w_2-2w\delta_2)A_2+(1-\alpha^2)A_3(w_3-2w\delta_3)\}]\exp(2k-4\delta)$$
$$+\surd(1+\theta^2)\surd(1-\alpha^2)\{B_2w_3-w_2B_3+4w(\delta_2B_3-\delta_3B_2)\}\exp(k-2\delta)$$
$$+2(1+\theta^2)\delta_2A_2+2(1-\alpha^2)\delta_3A_3-\Delta A=0 \tag{2.17}$$

where

$$\Delta\beta=(1+\theta^2)\beta_{22}+(1-\alpha^2)\beta_{33}+\theta\beta_2-\alpha\beta_3$$
$$P=\Delta w-4(1+\theta^2)w_2\delta_2-4(1-\alpha^2)w_3\delta_3$$
$$Q=(1-w^2\exp(2k-4\delta))^{-1}.$$

Here and in what follows the lower suffixes 2 and 3 after an unknown function denote partial differentiation with respect to θ and α respectively.

3. The solution

The field equations (2.10) to (2.17) are simultaneous, nonlinear partial differential equations of second order. It is very difficult to obtain a general solution of these equations. To simplify the problem mathematically, let us suppose that the potential functions A and B are related as follows

$$\surd(1+\theta^2)B_2=-\surd(1-\alpha^2)A_3$$
$$\surd(1-\alpha^2)B_3=\surd(1+\theta^2)A_2. \tag{3.1}$$

These relations between the potentials A and B simplify the field equation (2.15) into a form

$$\frac{\partial}{\partial\theta}[\exp(2\delta-k)]\frac{\partial}{\partial\alpha}[\exp(2\delta-k)]=\frac{\partial w}{\partial\theta}\frac{\partial w}{\partial\alpha}.$$

A particular solution of this equation is

$$w=\exp(2\delta-k)+C \tag{3.2}$$

where C is an arbitrary constant. The condition $B_{23}=B_{32}$ implies that

$$\Delta A=0. \tag{3.3}$$

And it is easy to see that

$$\Delta B=0. \tag{3.4}$$

Now the equation (2.16) is satisfied identically. The equation (2.11) implies that

$$\Delta\beta=0, \tag{3.5}$$

and the equation (2.14) gives a result of the form

$$C^2\exp(2k-2\delta)\{\Delta\delta-2(1+\theta^2)\delta_2^2-2(1-\alpha^2)\delta_3^2\}=-2\{(1+\theta^2)A_2^2+(1-\alpha^2)A_3^2\}. \tag{3.6}$$

Let us substitute

$$X=\mathrm{e}^{-2\delta},$$

then

$$\Delta X=\frac{4\,\mathrm{e}^{-2k}}{C^2}[(1+\theta^2)A_2^2+(1-\alpha^2)A_3^2]. \tag{3.7}$$

The equations (3.3), (3.4) and (3.5) are linear partial differential equations of second order. The author (Patel 1979) has solved these equations. The solution of equation (3.5) is

$$\begin{aligned}\beta=\Bigg[C_0\Bigg(1-\sum_{s=1}^{\infty}\frac{n^2(2^2-n^2)(4^2-n^2)\dots\{(2s-2)^2-n^2\}}{(2s)!}\alpha^{2s}\Bigg)\\
+C_1\alpha\Bigg(1+\sum_{s=1}^{\infty}\frac{(1^2-n^2)(3^2-n^2)\dots\{(2s-1)^2-n^2\}}{(2s+1)!}\alpha^{2s}\Bigg)\Bigg]\\
\times\Bigg[D_0\theta^{-n}\Bigg(1+n\sum_{s=1}^{\infty}\frac{(-1)^s(n+s+1)(n+s+2)\dots(n+2s-1)}{s!}\Big(\frac{1}{2\theta}\Big)^{2s}\Bigg)\\
+D_1\theta^n\Bigg(1+n\sum_{s=1}^{\infty}\frac{(-1)^{s+1}(s+1-n)(s+2-n)\dots(2s-1-n)}{s!}\Big(\frac{1}{2\theta}\Big)^{2s}\Bigg)\Bigg]\end{aligned} \tag{3.8}$$

where C_0, C_1, D_0, D_1 are arbitrary constants and n is a constant parameter of the family. The electromagnetic potentials A and B can be obtained by solving equations (3.3) and (3.4). A class of solutions is known but only physically meaningful solution of these equations will be considered.

$$A=\frac{\sigma\theta\alpha}{1+\theta^2-\alpha^2} \tag{3.9}$$

where σ, a constant, is the solution of the equation (3.3), which is the electric potential due to the distribution of electric charge over an oblate spheroidal source at the origin. Then

$$B=\frac{\sigma\sqrt{(1+\theta^2)}\sqrt{(1-\alpha^2)}}{1+\theta^2-\alpha^2} \tag{3.10}$$

which is the magnetic potential due to the rotation of an electrically charged source.

Now equation (3.7) becomes

$$\Delta X=\frac{4\sigma^2(\theta^2+\alpha^2)\,\mathrm{e}^{-2k}}{(1+\theta^2-\alpha^2)^2C^2}. \tag{3.11}$$

This is an inhomogeneous linear partial differential equation of second order. Its complementary function, as obtained by the author (Patel 1978) is,

$$\begin{aligned} X=1-\Big[&C_0'\Big(1+\sum_{s=1}^{\infty}\frac{-m^2(2^2-m^2)(4^2-m^2)\ldots\{(2s-2)^2-m^2\}}{(2s)!}\alpha^{2s}\Big)\\ &+C_1'\alpha\Big(1+\sum_{s=1}^{\infty}\frac{(1^2-m^2)(3^2-m^2)\ldots\{(2s-1)^2-m^2\}}{(2s+1)!}\alpha^{2s}\Big)\Big]\\ &\times\Big[D_0'\theta^{-m}\Big(1+m\sum_{s=1}^{\infty}\frac{(-1)^s(m+s+1)(m+s+2)\ldots(m+2s-1)}{s!}\Big(\frac{1}{2\theta}\Big)^{2s}\Big)\\ &+D_1'\theta^{m}\Big(1+m\sum_{s=1}^{\infty}\frac{(-1)^{s+1}(s+1-m)(s+2-m)\ldots(2s-1-m)}{s!}\Big(\frac{1}{2\theta}\Big)^{2s}\Big)\Big] \end{aligned} \tag{3.12}$$

and a particular integral is

$$X=\frac{2\sigma^2\,\mathrm{e}^{-2k}}{C^2(1+\theta^2-\alpha^2)}. \tag{3.13}$$

The general solution of the equation (3.11) is the addition of complementary function (3.12) and particular integral (3.13), hence

$$X=\mathrm{e}^{-2\delta}=\mathrm{CF}+\mathrm{PI}. \tag{3.14}$$

Now it is easy to see that rest of the field equations are satisfied identically.

Let us examine the behaviour of the solution at spatial infinity. In case of oblate spheroidal coordinates, distance from the axis of symmetry, r and distance from origin, R are given by the expressions

$$r=a\sqrt{(1+\theta^2)}\sqrt{(1-\alpha^2)} \qquad \text{and } R=a\sqrt{(1+\theta^2-\alpha^2)}. \tag{3.15}$$

Here A and B both vanish at least as R^{-1}. As $R\to\infty$, β in equation (3.8) must tend to zero so we choose

$$n>0 \qquad \text{and } D_1=0. \tag{3.16}$$

Also for the metric (2.1) to be asymptotically flat, the following conditions are to be satisfied.

$$D_1'=0 \qquad m>0. \tag{3.17}$$

Thus

$$e^{-2\delta}=\frac{2\sigma^2 e^{-2k}}{C^2(1+\theta^2-\alpha^2)}+1-\left[C_0'\left(1-\sum_{s=1}^{\infty}\frac{m^2(2^2-m^2)(4^2-m^2)\ldots\{(2s-2)^2-m^2\}}{(2s)!}\alpha^{2s}\right)\right.$$
$$\left.+C_1'\alpha\left(1+\sum_{s=1}^{\infty}\frac{(1^2-m^2)(3^2-m^2)\ldots\{(2s-1)^2-m^2\}}{(2s+1)!}\alpha^{2s}\right)\right]$$
$$\times\left[D_0'\theta^{-m}\left(1+\sum_{s=1}^{\infty}\frac{(-1)^s m(m+s+1)(m+s+2)\ldots(m+2s-1)}{s!}\left(\frac{1}{2\theta}\right)^{2s}\right)\right] \tag{3.18a}$$

$$\beta=\left[C_0\left(1-\sum_{s=1}^{\infty}\frac{n^2(2^2-n^2)\ldots\{(2s-2)^2-n^2\}}{(2s)!}\alpha^{2s}\right)\right.$$
$$\left.+C_1\alpha\left(1+\sum_{s=1}^{\infty}\frac{(1^2-n^2)(3^2-n^2)\ldots\{(2s-1)^2-n^2\}}{(2s+1)!}\alpha^{2s}\right)\right]$$
$$\times\left[D_0\theta^{-n}\left(1+n\sum_{s=1}^{\infty}\frac{(-1)^s(n+s+1)(n+s+2)\ldots(n+2s-1)}{s!}\left(\frac{1}{2\theta}\right)^{2s}\right)\right] \tag{3.18b}$$

$$w=e^{2\delta-k}+C \tag{3.18c}$$

is a class of solutions representing the exterior gravitational field of a rotating stationary charged oblate spheroid.

4. Physical interpretation

Fields satisfying equations (2.4)–(2.7) are broadly divided into two categories (i) null fields and (ii) non-null fields. In the case of a non-null field the electromagnetic energy momentum tensor E_{ij} possesses two–to–two equal and opposite eigenvalues (i.e. λ, $-\lambda$, λ, $-\lambda$). When all the eigenvalues are zero and nonvanishing F_{ij} satisfying equation (2.7) exist, the field is called a null electromagnetic field (Rao and Pandey 1962). In view of equations (3.2) and (3.5), the nonvanishing components of mixed Einstein tensors G^i_j and Maxwell tensors F_{ij} are

$$G^0_0=-G^1_1=-CG^0_1=(1/C)G^1_0=\frac{2\sigma^2}{Ca^2(1+\theta^2-\alpha^2)^2}\exp(-2\beta-k) \tag{4.1}$$

$$F_{02}=-CF_{12}=\frac{\sigma\alpha(1-\theta^2-\alpha^2)}{(1+\theta^2-\alpha^2)^2} \tag{4.2a}$$

$$F_{03}=-CF_{13}=\frac{\sigma\theta(1+\theta^2+\alpha^2)}{(1+\theta^2-\alpha^2)^2}. \tag{4.2b}$$

Therefore the four eigenvalues of the Einstein tensor defined by an equation

$$|G^i_j-\lambda g^i_j|=0 \tag{4.3}$$

vanish identically. The eigenvector associated with $\lambda_{(0)}$ no longer remains time-like but becomes null. The other eigenvectors are space like. The resulting space–time describes a null electromagnetic field with photons streaming along the ϕ direction with the fundamental velocity. Hence it follows that the line element (2.1) with the relations (3.2) and (3.5) is always compatible with a null electromagnetic field. Also the physical significance of the field can be studied by examining nature of an invariant $R_{ij}R^{ij}$. The nonzero components of the Riemann curvature tensor satisfy the relations

$$R_{0202} = -CR_{0212} = C^2R_{1212} \tag{4.4a}$$

$$R_{0303} = -CR_{0313} = C^2R_{1313} \tag{4.4b}$$

$$R_{0203} = -CR_{0213} = -CR_{0312} = C^2R_{1213} \tag{4.4c}$$

and nonzero components of the Ricci tensor are related by the relations

$$R_{00} = -CR_{01} = C^2R_{11}. \tag{4.5}$$

Hence by direct calculation it can be shown that the invariant

$$R_{ij}R^{ij} = 0. \tag{4.6}$$

The Petrov classification of Weyl's conformal curvature tensor plays a fundamental role in an invariant theory of gravitational radiation. For the Petrov classification of the Weyl's conformal curvature tensor of the metric (2.1), let us introduce the Pirani (1957) scheme of 6-dimension formalism. 6-dimension pseudo-Euclidean space is introduced in which vectors are, just bivectors (skew tensors) in a local tangent Minkowski space defined by the tetrad

$$\lambda^i_{(0)} = (-\mathrm{e}^{-\delta}, -(\mathrm{e}^{\delta-k} + C\mathrm{e}^{-\delta}), 0, 0)$$

$$\lambda^i_{(1)} = (0, \mathrm{e}^{\delta-k}, 0, 0)$$

$$\lambda^i_{(2)} = \left(0, 0, \frac{\sqrt{(1+\theta^2)}}{a\sqrt{(\theta^2+\alpha^2)}}\mathrm{e}^{-\beta}, 0\right)$$

$$\lambda^i_{(3)} = \left(0, 0, 0, \frac{\sqrt{(1-\alpha^2)}}{a\sqrt{(\theta^2+\alpha^2)}}\mathrm{e}^{-\beta}\right).$$

Physical components of Weyl's conformal curvature tensor C_{hijk} go over to the components of the symmetric 6-tensor

$$C_{\mu\nu} = (\mu, \nu = 1, 2, \dots 6)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -p & -q & 0 & -q & p \\ 0 & -q & p & 0 & p & q \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q & p & 0 & p & q \\ 0 & p & q & 0 & q & -p \end{bmatrix}$$

where

$$p = \frac{\mathrm{e}^{-2\beta}}{2a^2(\theta^2+\alpha^2)}\left[(1+\theta^2)(\delta_{22} - 2\delta_2^2 - 2\delta_2\beta_2) - (1-\alpha^2)(\delta_{33} - 2\delta_3^2 - 2\delta_3\beta_3) \right.$$
$$\left. + \frac{2+\theta^2-\alpha^2}{\theta^2+\alpha^2}(-\theta\delta_2 + \alpha\delta_3)\right]$$

$$q=\frac{\sqrt{(1+\theta^2)}\sqrt{(1-\alpha^2)}}{a^2(\theta^2+\alpha^2)}\left[\delta_{23}-2\delta_2\delta_3-\delta_2\beta_3-\delta_3\beta_2-\frac{1}{\theta^2+\alpha^2}(\alpha\delta_2+\theta\delta_3)\right]\mathrm{e}^{-2\beta}.$$

$$C_{\mu\nu}=\begin{bmatrix} M & N \\ N & -M \end{bmatrix},$$

then

$$P=M+\mathrm{i}N=\begin{bmatrix} 0 & 0 & 0 \\ 0 & -p-\mathrm{i}q & -q+\mathrm{i}p \\ 0 & -q+\mathrm{i}p & p+\mathrm{i}q \end{bmatrix}$$

where $\mathrm{i}=\sqrt{-1}$.

All three eigenvalues of the matrix P are equal and each is equal to zero. Also $P\neq 0$ but $P^2=0$; hence the canonical form of Weyl's conformal curvature tensor is of type $-N$.

These results characterise the null electromagnetic radiation. Lichnerowicz (1956) interpreted the null electromagnetic fields as a fluid distribution of photons with null geodesics as lines of flow.

5. Conclusion

The metric obtained in § 3 above is nonsingular for $\theta>\frac{1}{2}$ and $-1<\alpha<1$. The singularity regions of the solution (3.18) are (i) axis of summetry $\alpha=+1$ or -1, and (ii) $\theta\leqslant\frac{1}{2}$, that is an oblate spheroid of thickness less or equal to $\boldsymbol{a}$ through the axis of symmetry and of radius less or equal to $(\sqrt{5}/2)\boldsymbol{a}$ in the equatorial plane. Also when $\sigma=0$, that is when electromagnetic fields are switched off the solution (3.18) becomes a class of axially symmetric stationary asymptotically flat exact solutions of Einstein's vacuum field equations obtained by the author (Patel 1978). Hence the source of the metric so obtained is the charged rotating oblate spheroid $\theta\geqslant\frac{1}{2}$ or semi-infinite line source along the axis of symmetry as in the NUT solution (Bonnor 1969).

Acknowledgments

The author's grateful thanks are due to Professor J K Rao for his valuable guidance and suggesting improvements in this paper. Also the author is grateful to the referee for many helpful comments.

References

Banerji S 1968 *J. Phys. A: General Phys.* **1** 520
Blackett P M S 1947 *Nature* **159** 658
Bonnor W B 1961 *Z. Physik* **161** 439
—— 1969 *Proc. Camb. Phil. Soc.* **66** 145
Israel W and Wilson G A 1972 *J. Math. Phys.* **13** 865
J Krishna Rao and S N Pandey 1962 *Proc. Nat. Inst. of Sc. of India* **28**
Lichnerowicz A 1955 *Theries Relativistates de la gravitation et de l'Electromagnetism* (Paris: Masson et cie) 54
Mirsa M 1960 *Proc. Nat. Inst. Sc. India* **26**A 673

Patel L K 1975 *J. Aust. Math. Soc.* **19**(B), 140
Patel M D 1979 *J. Aust. Math. Soc.* **21**B
Perjes Z 1971 *Phys. Rev. Lett.* **27** 1668
Pirani F A E 1957 *Phys. Rev.* **105** 1089
Som M M and Raychaudhari A K 1968 *Proc. R. Soc.* **304**A 81
Synge J L 1960 *Relativity: the general theory* (North-Holland: Amsterdam) p 369